Two-Stage Facility Location Games with Strategic Clients and Facilities

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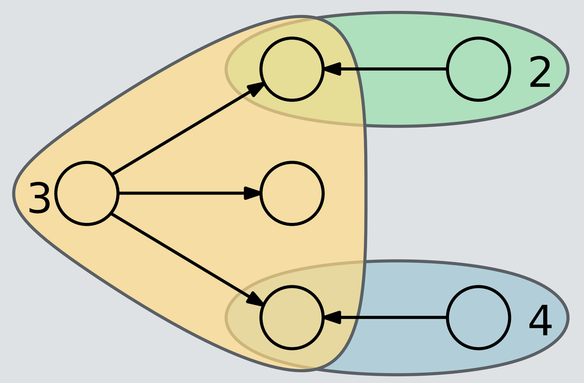
Our Model (2-FLG)

- k facility agents compete for client weight (= buying power)
- n client agents aim to use facilities that have the lowest possible load

Host graph

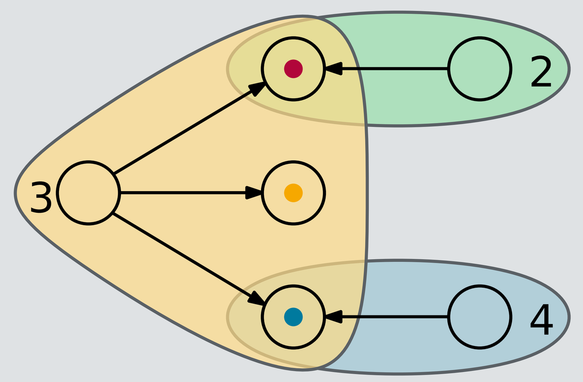
Directed graph with integer-weighted client fixed to some nodes



Each client has a maximum shopping distance of 1 (colored areas)

Stage 1

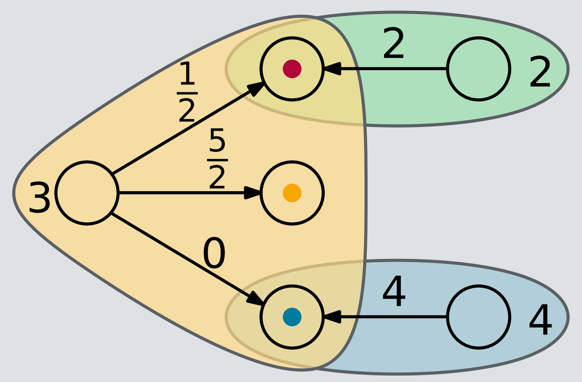
Facility agents place one facility (•) each on any node



Placement on an node occupied by another facility/client is possible.

Stage 2

Client agents distribute their weight among facilities in shopping range



Clients aim to minimize the maximum load of their visited facilities

Balanced by the left client, the red and yellow facilities receive a load of $\frac{5}{2}$ each. The blue facility receives a load of 4.

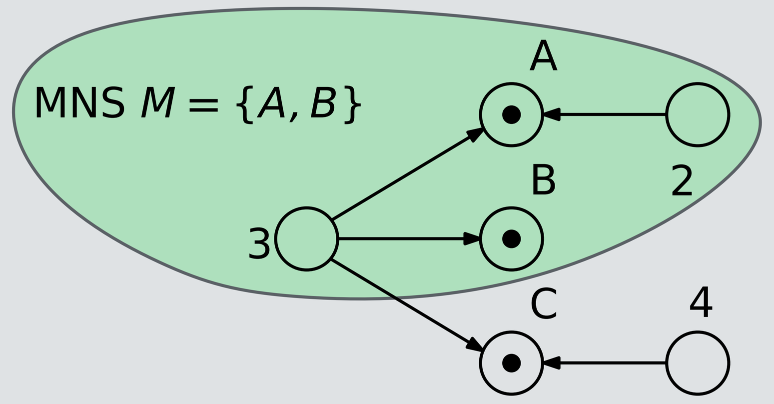
Client equilibria

Theorem: Client equilibria exist for all placements of facilities. For each placement the facility utilities are the same in all client equilibria.

Minimum Neighborhood Set (MNS): Let $w(A_S(M))$ be the total weight of the clients, that can reach any facility in the set of facilities M. A MNS is the non-empty set M of facilities for which $\frac{w(A_S(M))}{|M|}$ is minimal. A MNS is computable in polynomial time using flows.

Theorem: In a client equilibrium, each facility in an MNS M receives a load of $\frac{w(A_S(M))}{|M|}$.

Example of an MNS



Weight of clients who can reach any facility in M: $3 + 2 = 5$

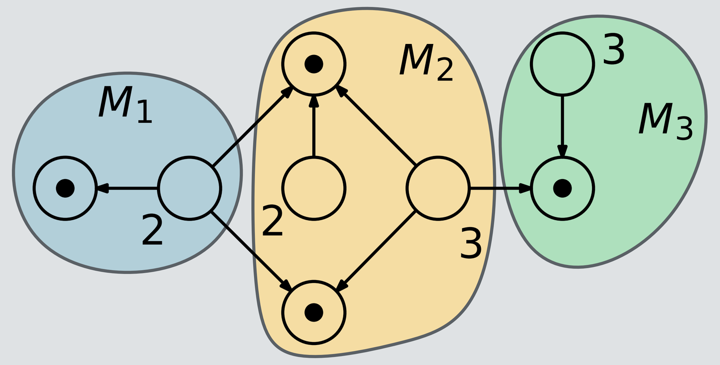
$\frac{w(A_S(M))}{|M|} = \frac{5}{2}$

Load Algorithm

While there are facilities left:

- Compute MNS
- Assign loads to included facilities (see theorem above)
- Remove included facilities and their reachable clients

Example Run



M_1 is a MNS with a ratio of 2. After removal of M_1 and its client, M_2 is an MNS with a ratio of $\frac{5}{2}$. Finally, only one facility and client remain for M_3.

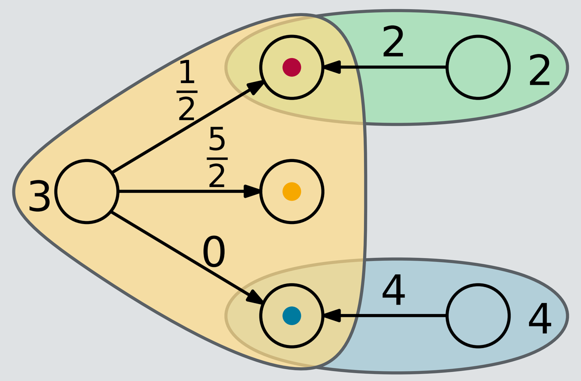
Subgame perfect equilibria (SPE)

A *subgame perfect equilbrium* needs to be an equilibrium for client agents and facility agents.

Theorem: Each instance of 2-FLG has an SPE.

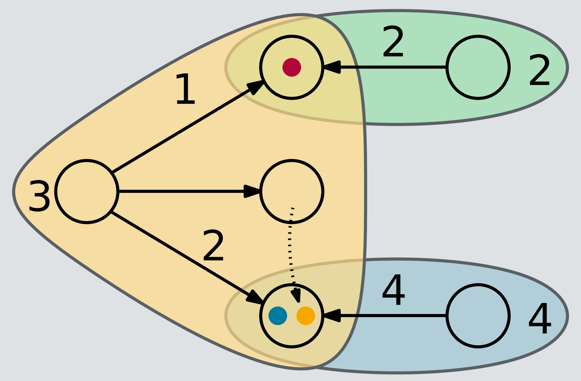
Proof Sketch: If a facility agent A changes her strategy and receives a utility of u after her move, no other facility utility A' decreases to a value $u' < u$ because of the strategy change of A. Hence, we can proof the statement via lexicographial potential function over the utilities of the facility agents. □

Original State



Facility utilities: $\frac{5}{2}$, $\frac{5}{2}$, 4

After Improving move



Facility utilities: 3, 3, 3

The utility of the blue facility does not decrease below the new utility of the moving facility.

Equilibrium Efficiency

We measure the efficency by the sum of facility utilities which is equivalent to the amount of client weight covered by at least one facility.

Theorem: Computing the socially optimal facility placement is NP-hard.

Efficiency Bounds

	lower bound	upper bound
Price of Anarchy and Price of Stability	$2 - \frac{1}{k}$	2

Open Questions

- How hard is the computation of an SPE?
- How fast do best response dynamics converge to an SPE?
- What happens when clients value distance and load?

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