Equilibria and Convergence in Fire Sale Games

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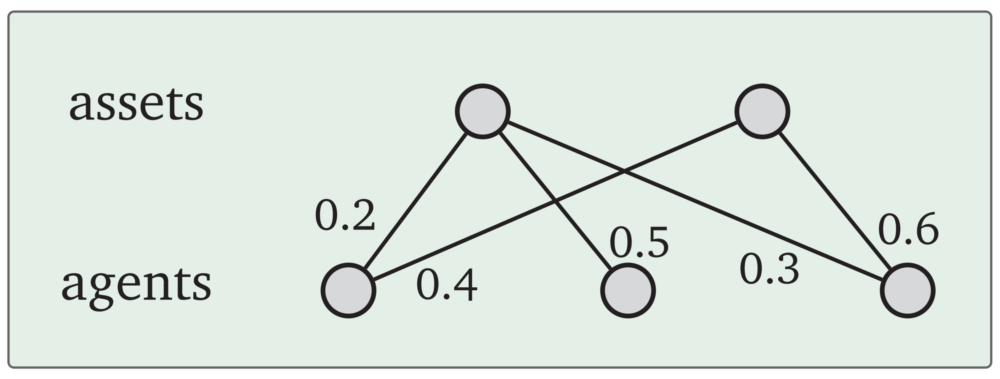
Model

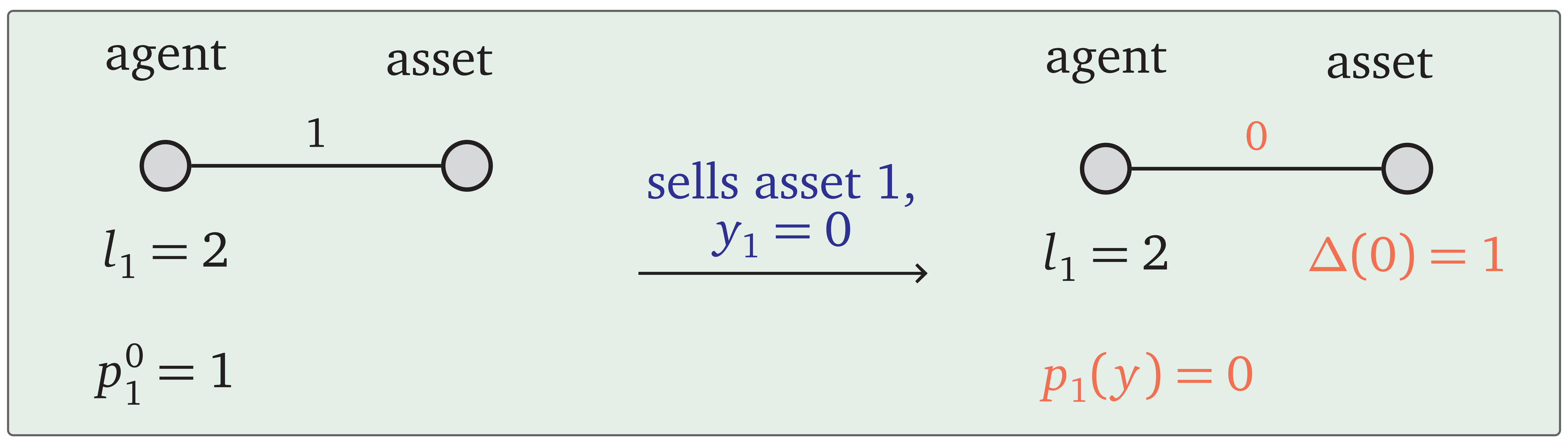


- Each agent $i \in N$ holds illiquid assets a_i^I, liabilities l_i and a share x_{ij} of asset $j \in M$
- Strategy $y_i \in [0,1]$ defines amount of assets i keeps with values $p_j(y)$

$$\text{total assets: } a_i(y) = a_i^I + y_i \sum_{j \in M} x_{ij} p_j(y)$$

- Assets are sold for a linear combination of pre- and post-sale prices. For implementation shortfall α, agent i gains

$$\Delta_i(y) = (1 - y_i) \sum_{j \in M} x_{ij}((1-\alpha)p_j^0 + \alpha p_j(y))$$

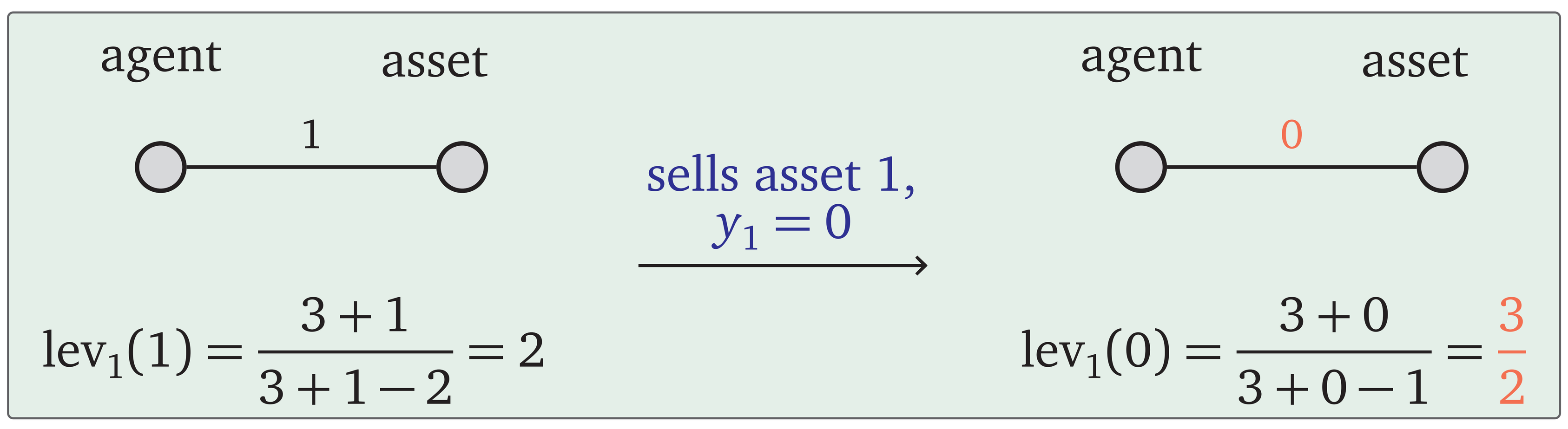


- Agents strategically sell (and buy) assets in order to maximize equity

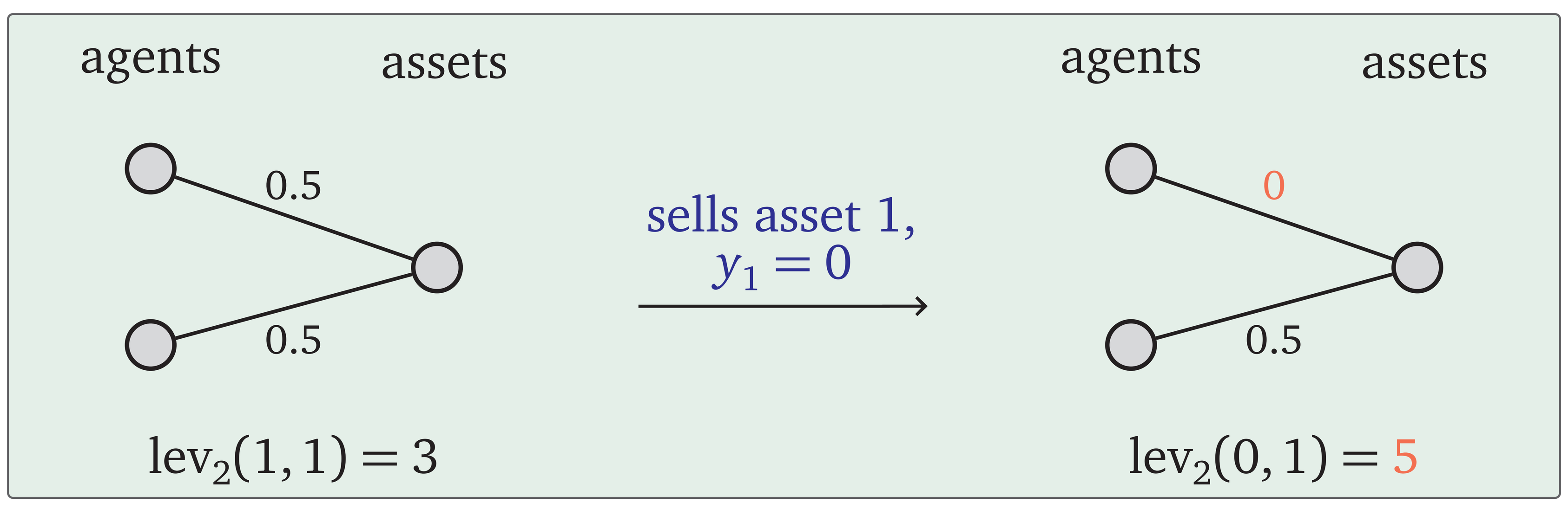
$$e_i(y) = a_i(y) + \Delta_i(y) - l_i$$

- Leverage constraint λ limits the allowed ratio of total assets and equity

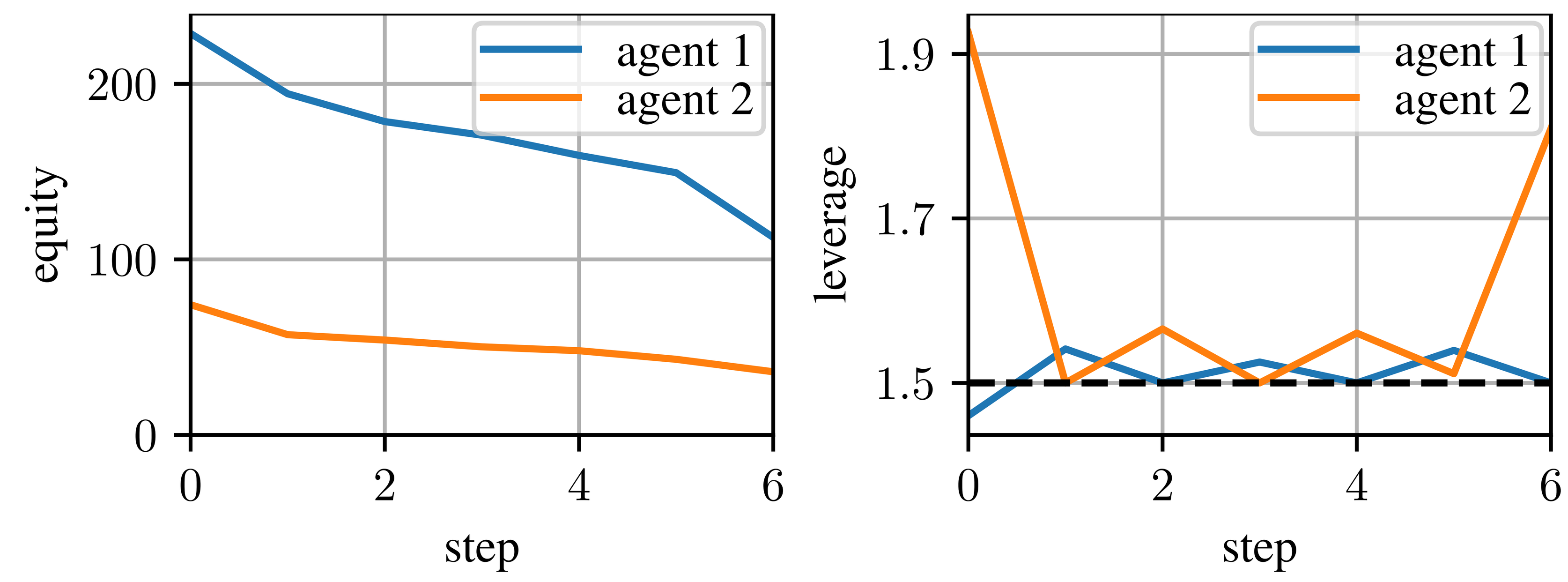
$$\text{lev}_i(y) = \frac{a_i(y)}{e_i(y)} = \frac{a_i(y)}{a_i(y) + \Delta_i(y) - l_i} \leq \lambda$$



- All agents must sell all assets if unable to satisfy the leverage constraint
- Fire-sale dynamics: Sales → price decrease → further sales …



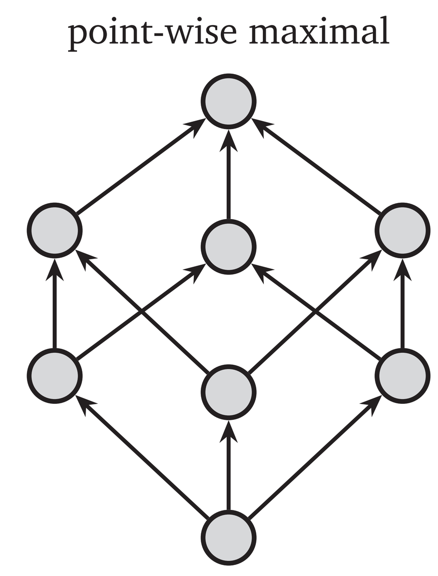
- Agents may drive each others' equity downward over multiple best responses:



In step 5 and 6, agent 2 has no strategy to fulfill the leverage constraint $\text{lev}_2(y) \leq 1.5$.

Results

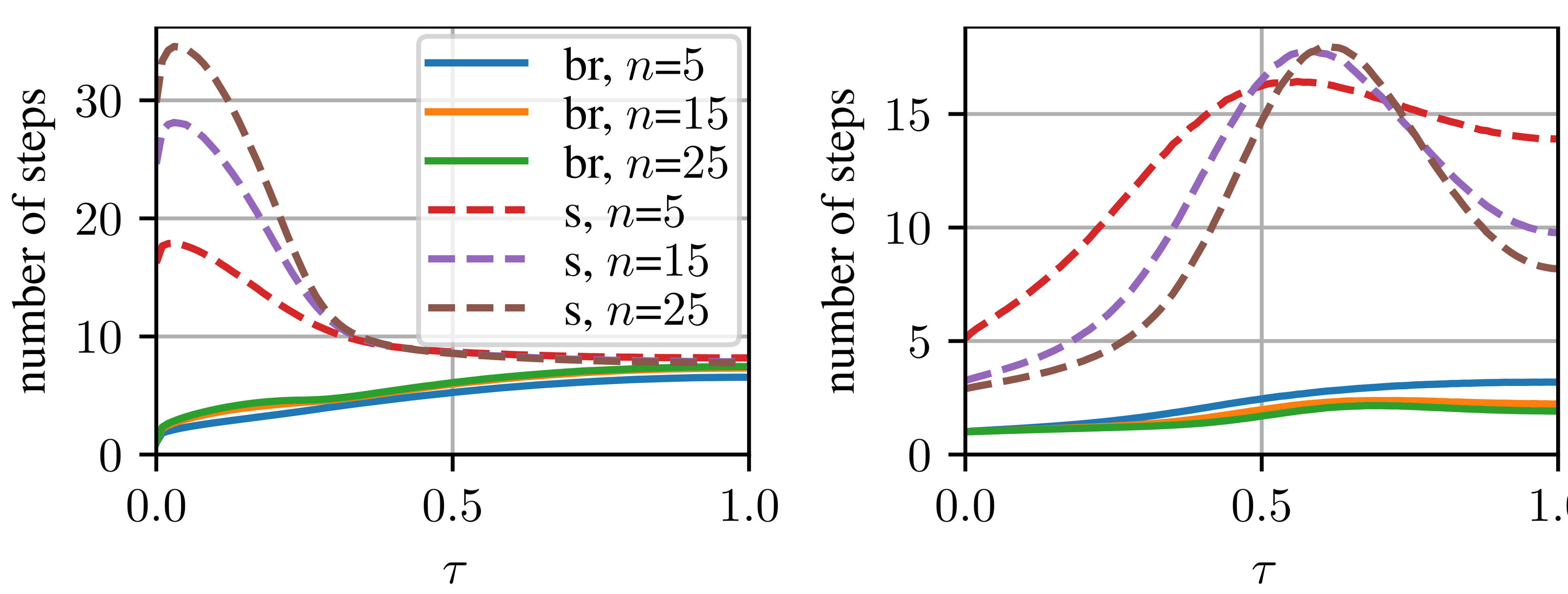
- For $\alpha = 1$, the best-response function is monotonic.
- Nash equilibria exist and form a lattice.
- Starting with no sales, the best-response dynamics converges to the point-wise maximal equilibrium.



- The best-response dynamics is acyclic for two players.
- Reaches an $(\text{poly}(x_{max}) \cdot \epsilon)$-approximate equilibrium after n/ϵ steps.
- In the simplified best-response dynamics the agents neglect their own impact on prices. The dynamics converges to the same equilibrium but is less computational demanding.

Convergence Time Experiments

We compare the convergence time of best responses and simplified best responses for two parameter sets for various degrees of diversification:



Open Questions

- Equilibria, dynamics for non-even sales?
- Dynamics for concave price impact?
- Bailout via asset transfers to stabilize the network?

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