Strategic Facility Location with Clients that Minimize Total Waiting Time

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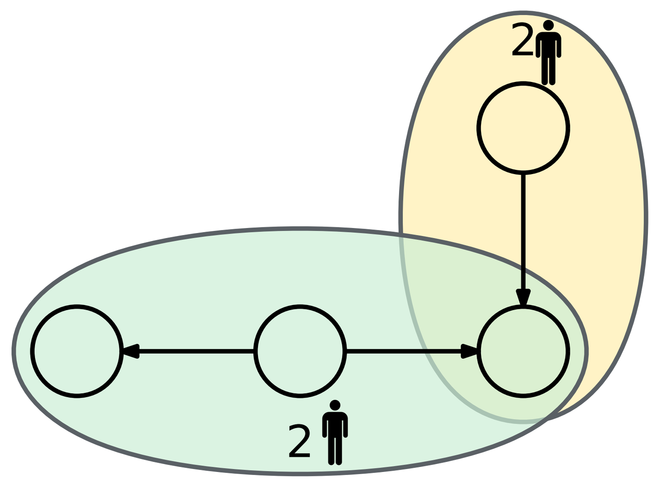
Our Model (Min-2-FLG)

- k facility agents compete for client weight (= buying power)
- n client agents aim to use facilities that have the lowest possible load

Host Graph

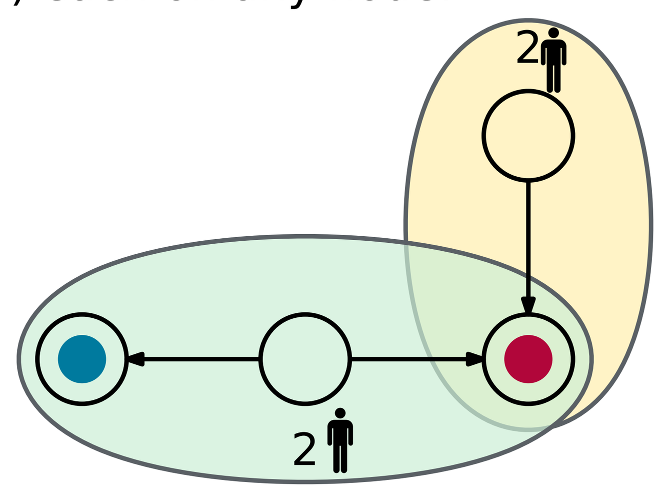
Directed graph with weighted clients fixed to some nodes.



Clients a have maximum shopping range of 1 (colored area).

Stage 1

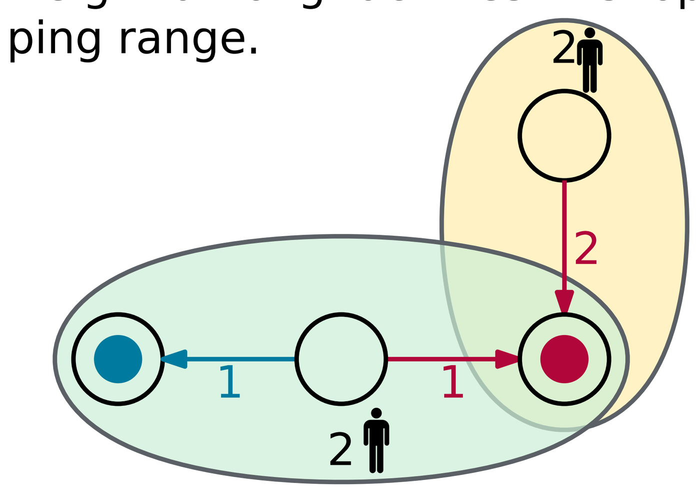
Facility agents place one facility (•) each on any node.



Placement on an already occupied is possible.

Stage 2

Client agents distribute their weight among facilities in shopping range.



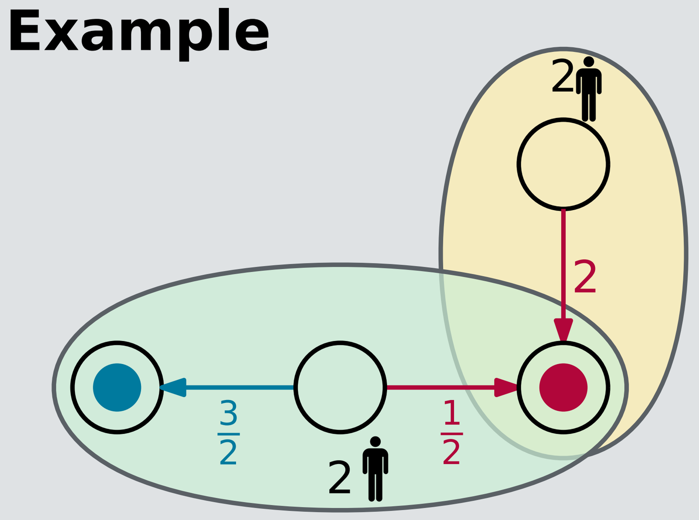
Client v aims to minimize the experienced $cost(v) =$

$$\sum_{f \in \text{Facilities}} \text{Weight}(v, f) \cdot \text{Load}(f)$$

Client Equilibrium

- Client game = atomic splittable congestion game
- Exactly one equilibrium (Bhaskar, Fleischer, Hoy, Huang, 2015)
- Computable in polynomial time (Harks, Timmermans, 2021)

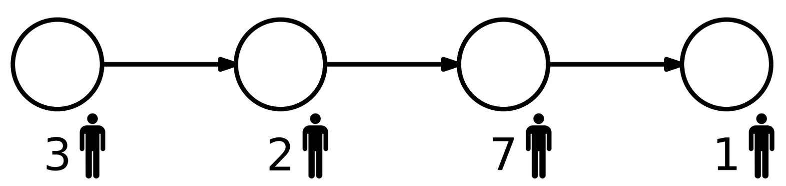
Client Equilibrium Example



The client equilibrium for the example instance is $\frac{3}{2} \cdot \frac{3}{2} + \frac{1}{2} \cdot \frac{5}{2} = \frac{7}{2}$. The bottom clients shifts some weight to the red facility to lower congestion for the rest of her weight.

Facility Equilibrium (Subgame Perfect Equilibrium)

Not in all instances. Example for two facility agents:



Theorem: Existence is NP-hard to compute (reduction from Independent Set).

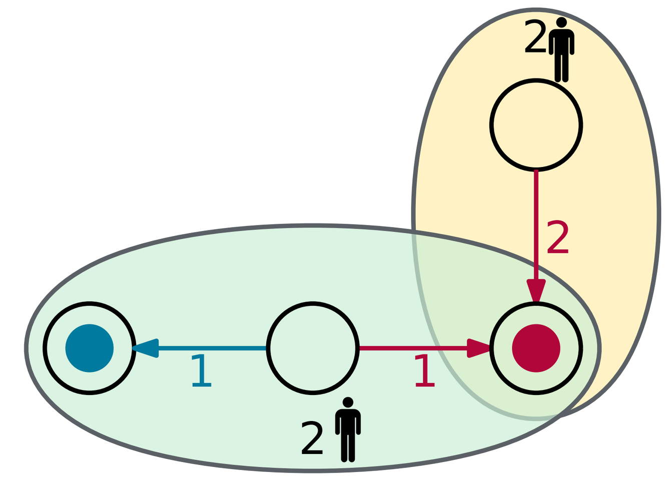
Contrast to guaranteed existence for nonatomic client stage, where clients minimize maximum weight of visited facilities (Krogmann, Lenzner, Molitor, Skopalik, 2021)

Facility Equilibrium Efficiency

Price of Anarchy and Price of Stability bounded by $2 - \frac{1}{k}$ and 2 (Krogmann, Lenzner, Molitor, Skopalik, 2021)

Write your Feedback and Suggestions here:

Uniform-2-FLG (Helper for Approximation)



Clients distribute their weight uniformly among all facilities in shopping range.

Facility game is an exact potential game (Rosenthal, 1973).

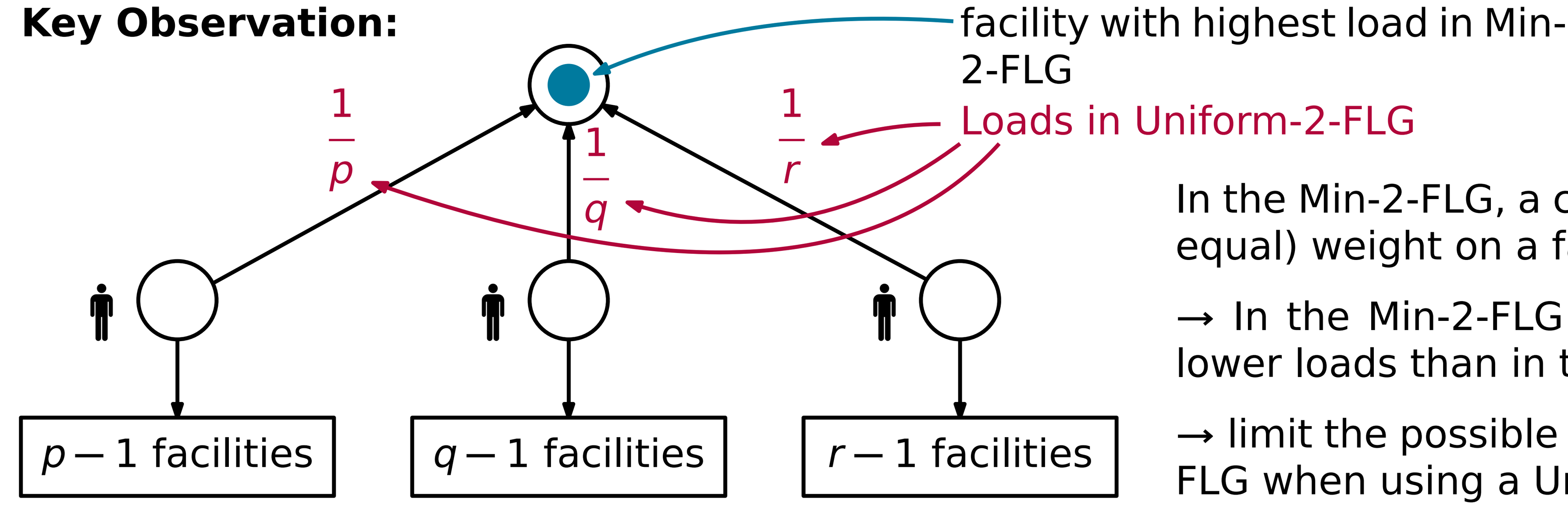
Theorem: A $(1+\epsilon)$-approximate facility equilibrium is reached in polynomial time by following best responses in $\mathcal{O}\left(\frac{1}{\epsilon} n^2 \log n\right)$ steps.

Theorem: Computing an exact facility equilibrium is PLS-complete (reduction from LocalMaxCut).

Relation between Min-2-FLG and Uniform-2-FLG

Theorem: A $(1+\epsilon)$-approximate facility equilibrium in the Uniform-2-FLG is a $(3+2\epsilon)$-approximate facility equilibrium in the Min-2-FLG.

Key Observation:



In the Min-2-FLG, a client will always put more (or equal) weight on a facility with a smaller load.

→ In the Min-2-FLG the blue facility will receive lower loads than in the Uniform-2-FLG.

→ limit the possible gain of a facility in the Min-2-FLG when using a Uniform-2-FLG equilibrium

Computing an Approximate Facility Equilibrium

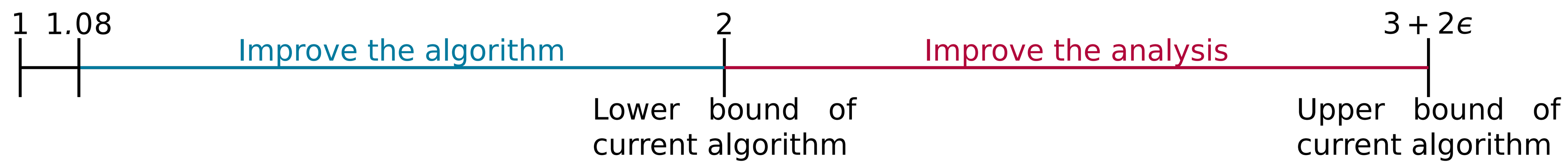
Theorem: A $(3+2\epsilon)$-approximate facility equilibrium in the Min-2-FLG can be computed in polynomial time.

Algorithm:

- start from an arbitrary state.
- while there is a best response in the Uniform-2-FLG with an improvement factor of $1+\epsilon$:
  - execute this best response

Open Questions

- Can the approximation be improved?



- What if client demand shifts over time?
- What if there are location-dependent costs? For clients? For facilities?

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